Dr. MPS Group of Institutions

BCA – 5th Semester

Question Bank: Design & Analysis of Algorithms (Jan – 2022)

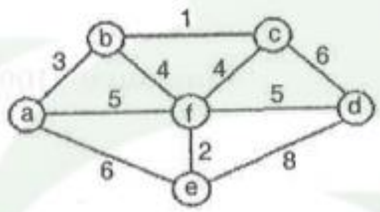
1. Explain briefly:

a) Define Big O notation, Big Omega and Big Theta Notation. Depict the same graphically and explain.

b) Explain Merge Sort Algorithm with an example.

2.

a) Give the Pseudo code for Prim's algorithm and apply the same to find the minimum spanning tree of the graph shown below :



(OR)

b) Explain the memory function method for the knapsack problem and give the algorithm.

3. a)

Compare the efficiency of the three algorithms :

(A) the brute-force algorithm. (B) this presorting-based algorithm, and

(C) the divide-and conquer algorithm.

(OR)

b) Apply Warshall's algorithm to find the transitive closure of the digraph defined by the following adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Prove that the time efficiency of Warshall's algorithm is cubic.

4. a) How Dynamic programming is used to solve Knapsack Problem?

b) Gove an example of sum-of-subset problem.

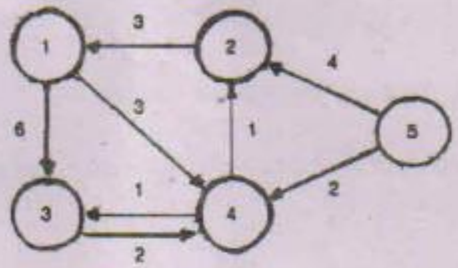
5. (a) (i) Prove that if $g(n)$ is $\Omega(f(n))$ then $f(n)$ is $O(g(n))$.

(ii) Discuss various methods used for mathematical analysis of recursive algorithms.

Or

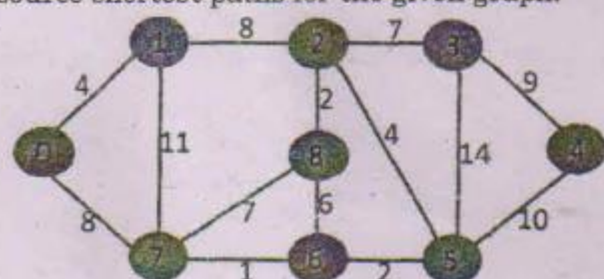
(b) Write the asymptotic notations used for best case, average case and worst case analysis of algorithms. Write an algorithm for finding maximum element in an array. Give best, worst and average case complexities.

6. (a) Explain Floyds – Warshall algorithm using dynamic programming. Trace the algorithm for the given example.



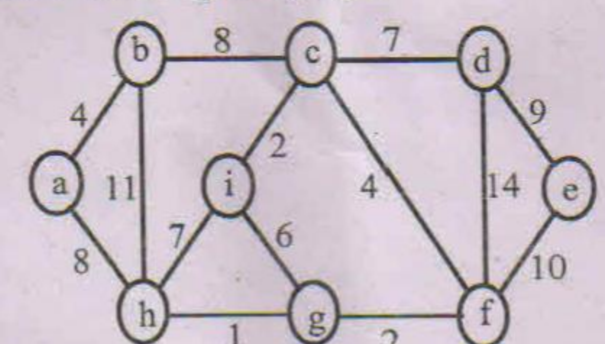
Or

(b) Explain how greedy approach is used in Dijkstra's algorithm for finding the single–source shortest paths for the given graph.



7.

(a) Apply the greedy technique to find the minimum spaning tree using Prim's algorithm for the given graph.



Or

(b) Explain the 4-Queen's problem using backtracking. Write the algorithms. Give the estimated cost for all possible solutions of 4-Queen's problem. Specify the implicit and explicit constraints.

8. a) Devise an algorithm to make for 1655 using the Greedy Strategy. The coins available are (1000, 500, 100, 50, 20, 10, 5).

b) What is divide and conquer strategy? Explain the binary search with the suitable example.

9. (a) Solve the following instance of the 0 / 1, knapsack problem given the knapsack capacity in $W = 5$ using dynamic programming and explain it.

Items	Weight	Value
1	4	10
2	3	20
3	2	15
4	5	25

Or

(b) Write the Huffman's Algorithm. Construct the Huffman's tree for the following data and obtain its Huffman's Code.

Character	A	B	C	D	E	–
Probability	0.5	0.35	0.5	0.1	0.4	0.2

10. a) Describe the backtracking solution to solve 8-queen problem.

b) Write an algorithm for quick sort and write its time complexity with example list are 5, 3, 1, 9, 8, 2, 4, 7.

11. Solve the following recurrence relations

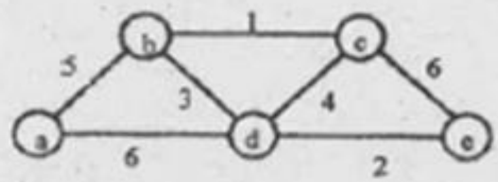
- $x(n) = x(n-1) + 5$ for $n > 1 x(1) = 0$
- $x(n) = 3x(n-1)$ for $n > 1x(1) = 4$
- $x(n) = x(n-1) + n$ for $n > 0$ $x(0) = 0$
- $x(n) = x(n/2) + n$ for $n > 1$ $x(1) = 1$ (solve for $n = 2^k$)
- $x(n) = x(n/3) + 1$ for $n > 1$ $x(1) = 1$ (solve for $n = 3^k$).

12. (a) Solve the all-pairs shortest-path problem for the digraph with the following weight matrix:

$$\begin{vmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{vmatrix}$$

Or

(b) Apply Kruskal's algorithm to find a minimum spanning tree of the following graph.



13. a) Solve the following instance of Knapsack problem by branch and bound algorithm.

Item	weight	profit
1	5	$40
2	7	$35
3	2	$18
4	4	$4
5	5	$10
6	1	$2

W=15

b) The Longest Increasing Subsequence (LIS) problem is to find the length of the longest subsequence of a given sequence such that all elements of the subsequence are sorted in increasing order. Write an algorithm using dynamic programming that determines the LIS of a string 'x'. For example, the length of LIS for {10, 22, 9, 33, 21, 50, 41, 60, 80} is 6 and LIS is {10, 22, 33, 50, 60, 80}.

14. (a) (i) Solve the following recurrence equation :

(1) $T(n) = T(n/2) + 1$, where $n = 2^k$ for all $k \geq 0$

(2) $T(n) = T(n/3) + T(2n/3) + cn$, where 'c' is a constant and 'n' is the input size.

(ii) Explain the steps involved in problem solving.

15. (a) (i) Given a matrix of order $M \times N$, and two coordinates (p, q) and (r, s), which represents the top-left and bottom-right of a sub-matrix of the matrix, $M \times N$, calculate the sum of elements present in the sub-matrix in $O(1)$ time using dynamic programming. Determine the optimal sub-structure and write an algorithm.

(ii) Prove that any algorithm that sorts by comparison, requires $\Omega(n \lg n)$ time.

Or

(b) (i) The longest common subsequence (LCS) is the problem of finding the longest subsequence that is present in the given two sequences in the same order but not necessarily contiguously. Write an algorithm using dynamic programming that determines the LCS of two strings, 'x' and 'y' and returns the string 'z'.

(ii) Prove that any algorithm that searches need to necessarily do $\Omega(\lg n)$ comparisons.